Peter McKenzie Armstrong

S y z y g i e s

Two–Part Takes on Wythoff's Matrix

Volume I

for piano solo

Opus 16a

Edition Ottaviano Petrucci

NOTES

Wythoff's matrix (2–dimensional array) is generated from two columns thereupon excluded from the matrix proper: the integers from 0; the floored products of their succesors and Phi (half of one more than the square root of 5, i.e., the "golden ratio"). Generation proceeds by line, applying the Fibonacci summation rule (each next term shall sum the two immediately preceding terms) to each integer pair:

0 1 | 1 2 3 5 ...
1 3 | 4 7 11 18 ...
2 4 | 6 10 16 26 ...
...

Taken to infinity, the result yields all positive integers, each exactly once. Most striking immediately is that each matrix line's first number is the lowest not previously named.

Since Fibonacci family sequences modulo–N yield finite cycles, and as most such cycles invite partitioning into complementary halves, it occurred to me to tease some Wythoff matrix lines accordingly into two–part melodic structures. For the number of lines to be so treated, I chose 15 –– recalling Sebastien Bach's Inventions and realizing that, within that work's C2–C6 range, 15 gives (for paired non–overlapping 1–to–3–octaves–wide voices) the total of range distributions available.

To realize Fibonacci proportions on a larger time scale, I harnessed them as the number of events in a given cycle, determining movement lengths. This was done indirectly, by specifying the modulus to be applied in each matrix–line generation. Pitch per cycle member was then determined via selective range and registration constraints.

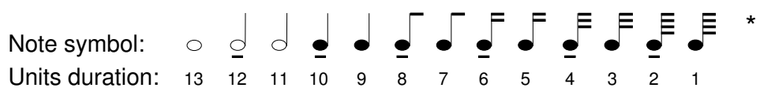
Duration for each cycle pitch was set to parallel one of: the pitch class (PC); its occurrence frequency (PC–OF); the completed serial interval (SI); its occurrence frequency (SI–OF). The result for each movement was then scored in four separate interpretations:

A1. Homophonic, dynamics parallel duration (whole=fff, 64th=ppp).
A2. Homophonic, with durations range inverted, originally longest notes becoming shortest and vice versa (whole=ppp, 64th=fff).
B1. Polyphonic, via retrogression of one part's order of durations.
B2. Polyphonic, again with the durations range inverted, as in A2.

Finally as systematic punctuation in all movements, a rest replaced each note having the least frequent duration value. The pairing of opposites –– especially the mutually inverse duration ranges in A1 vs A2 and B1 vs B2 –– inspired the work's title. In this volume movements with wider single–voice ranges are assigned the greater overall lengths; a subsequent volume will counter that relationship.

Notation of rhythm in this score needs special explanation. I have superimposed two ways to indicate note duration: time–proportional horizontal spacing; length–specific note glyphs. The former (with some whitespace slivers inserted to clear note/barline collisions) is intended to serve as the principal guide in performance.

The latter, nodding to tradition, is meant to enhance perspective on the grouping of details. It references a value spectrum based, not on reciprocal powers of 2 as historically, but on the consecutive counting numbers (to avoid masses of ties that a metered notation of non–metrical duration sequences would entail). Its vocabulary, seven note–graphics each alternatively modified via tenuto, is as follows:



*

In either notation, this music's few apparent "beats" are inadvertent, and simultaneity in its polyphonic textures is virtually absent. To a pianist possibly bent on performing the work, I grant that substantial rhythmic liberty will be needed to approach a controlled (repeatable) rendition.

* Graphic generated in XFig.

– PMA

MISCELLANY

[’]: Breath marks in either voice demarcate sections with equal numbers of events. Where they are synchronized between the voices, a barline precedes. If such demarcation is possible as well on a smaller scale, additional barlines intervene. In several polyphonic (B1/B2) textures, a barline may announce the only simultaneity.

Dur I: Durations range Inverted. Here in each syzygistic pair’s opposing portion (A2/B2) notes that in the first were shortest are now the longest, and vice versa: 1–>13, 2–>12, ..., 13>1. The pitch cycle is unaltered.

F[4–11]/L[3–9]: Index of the Fibonacci/Lucas member corresponding to the current cycle period. The F and L series, differing by seed pairs (0,1 vs 2,1), are the essential two from which Wythoff’s infinitely many others derive. Syzygies takes its periods from them alternately, via moduli ranging from 4 to 28657.

Octs ’[’[1] [2] [3] [4]’]’: Range specification. Of the 4 octaves C2–C6, those comprising the registers of lower and upper voices are named. Hyphens indicate a multi–octave single–voice range.

Per: Period, here the number of events in either voice. This value is half that of the full source integer–cycle period.

R: Retrograde, affecting both Pitch & Rhythm unless otherwise specified.

Volume (not scored) is to vary with note Duration, either directly (A1/B1) or inversely (A2/B2).

Playing time: ~10 minutes.

to Rebecca Raffaelli

Syzygies

Two-Part Takes on Wythoff's Matrix

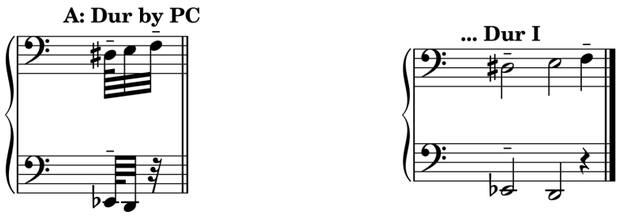
for piano solo

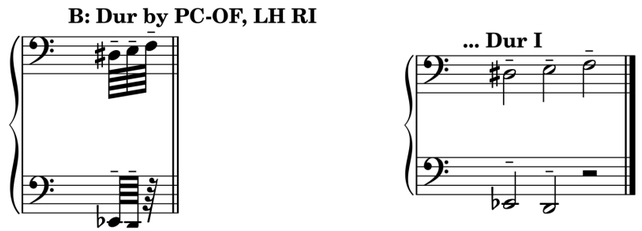
Peter McKenzie Armstrong

Opus 16a

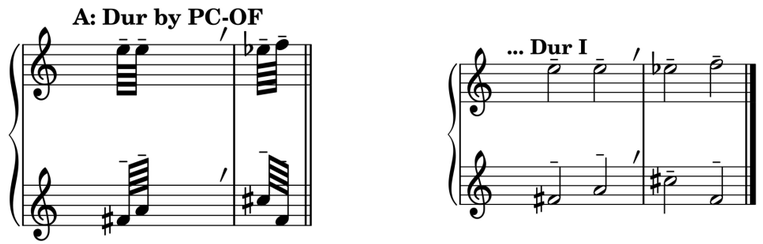
Tempo ♩ = 180

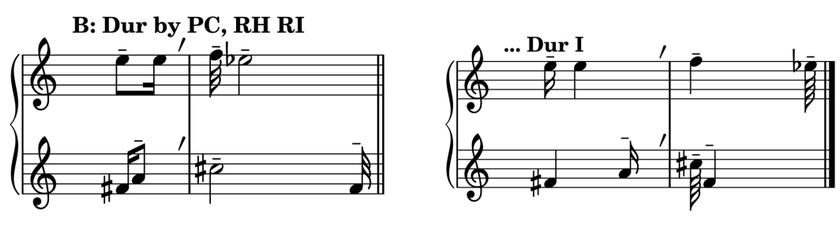
S1 Octs [1 2]
Seeds 0 1, Per 3 (F4)





S2 Octs [3 4]
Seeds 1 3, Per 4 (L3)





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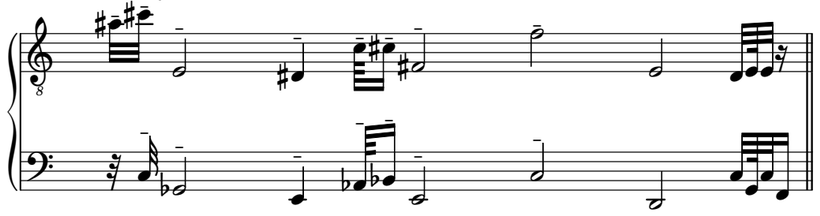
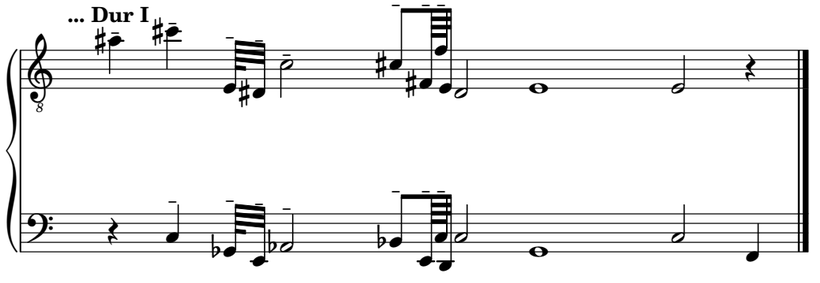
S3 Octs [1 2-3]
Seeds 2 4, Per 13 (F7)

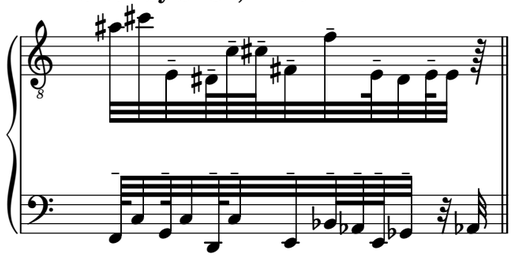
A: Dur by SI



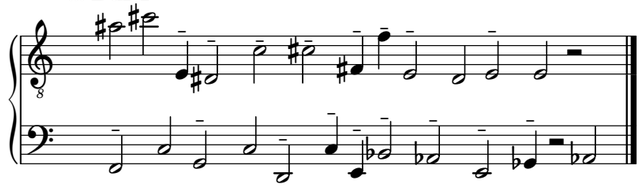
... Dur I



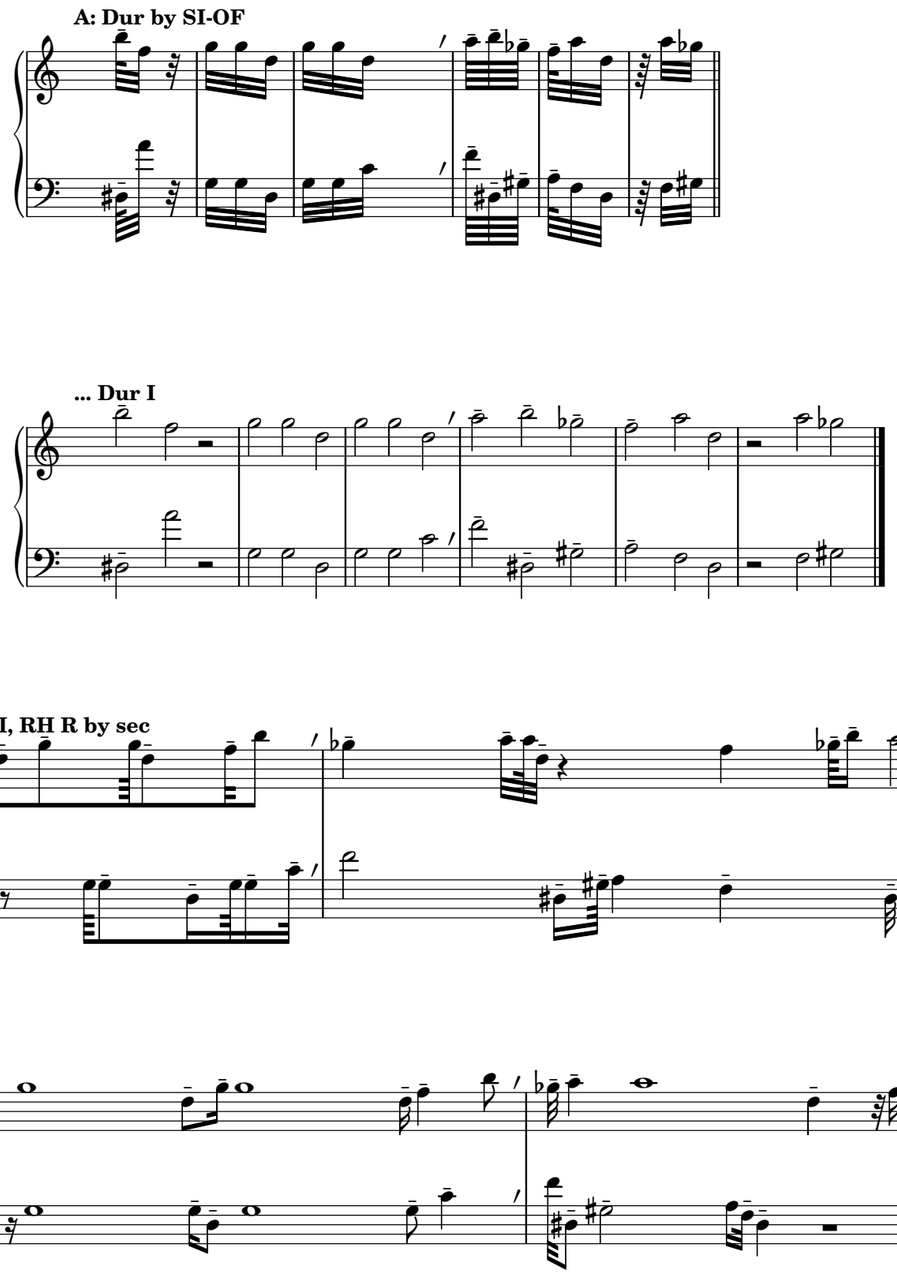
B: Dur by SI-OF, LH R



... Dur I



S4 Octs [2-3 4]
Seeds 3 6, Per 18 (L6)



S5 Octs [1 2---4]
Seeds 4 8, Per 55 (F10)

A: Dur by PC, LH Pitch R

... Dur I

B: Dur by SI-OF, LH Rhythm R

... Dur I

S6 **Octs [1---3 4]**
Seeds 5 9, Per 76 (L9)

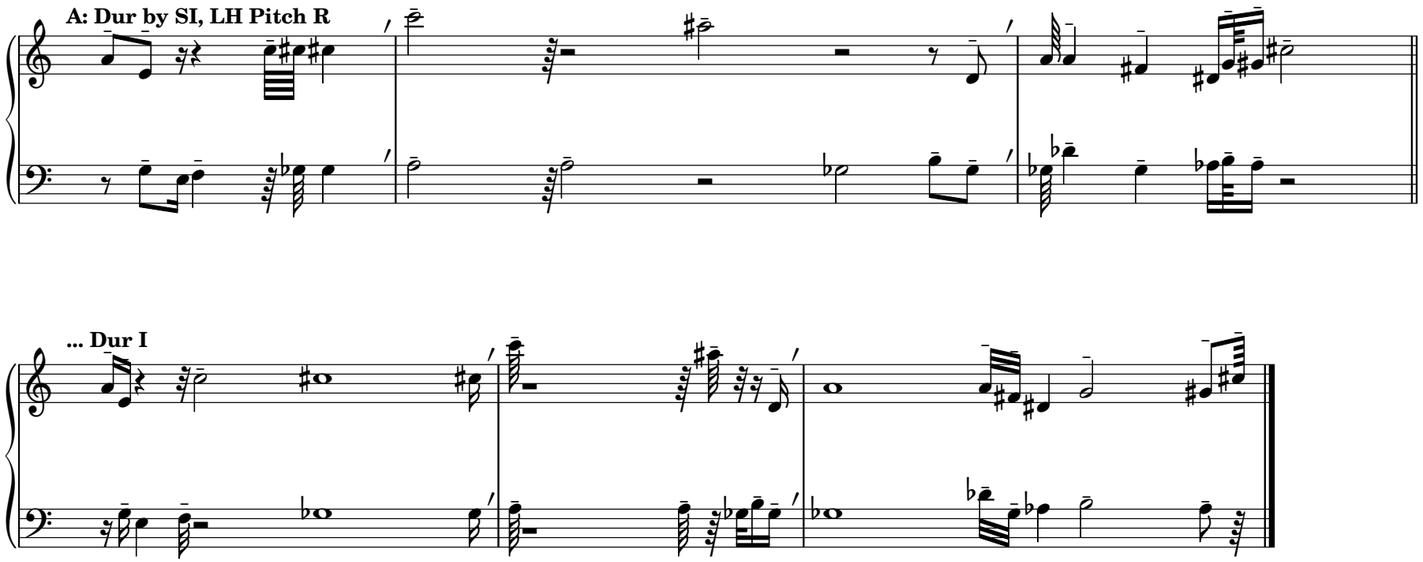
A: Dur by SI-OF, RH Pitch R

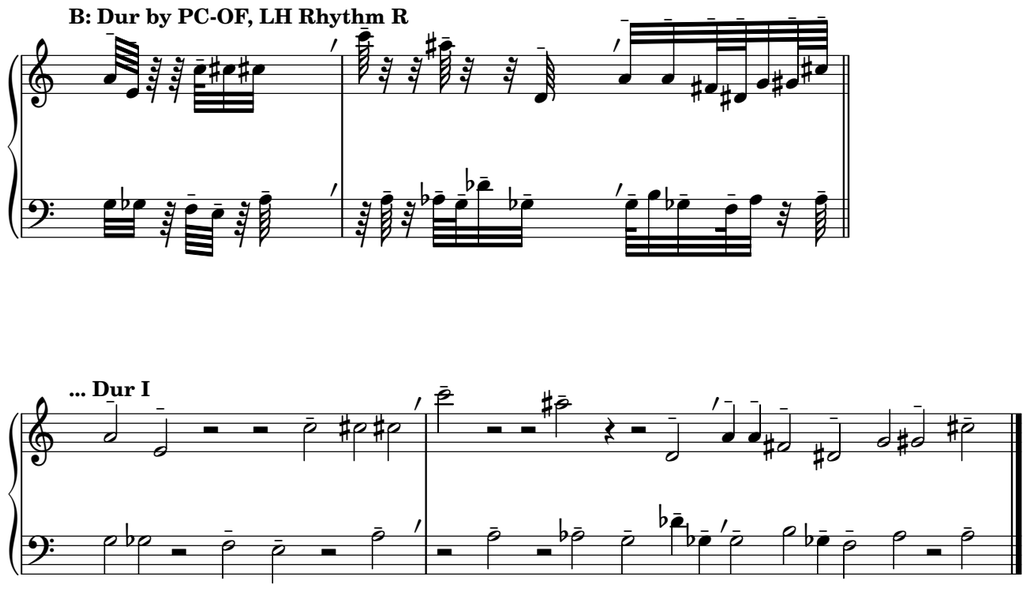
... Dur I

B: Dur by PC, LH Rhythm R

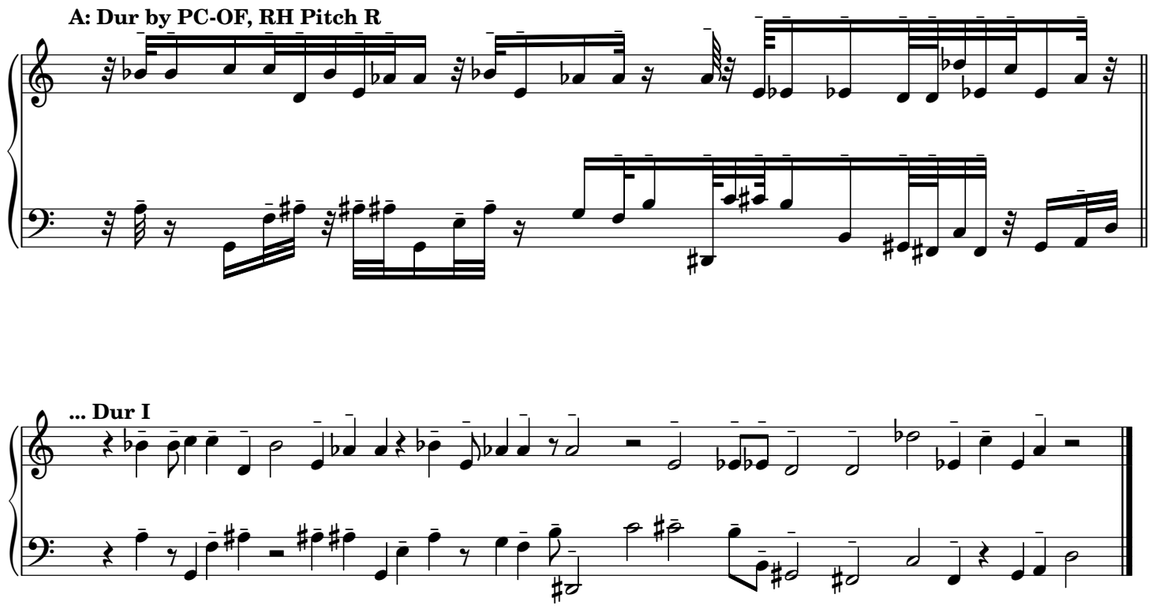
... Dur I

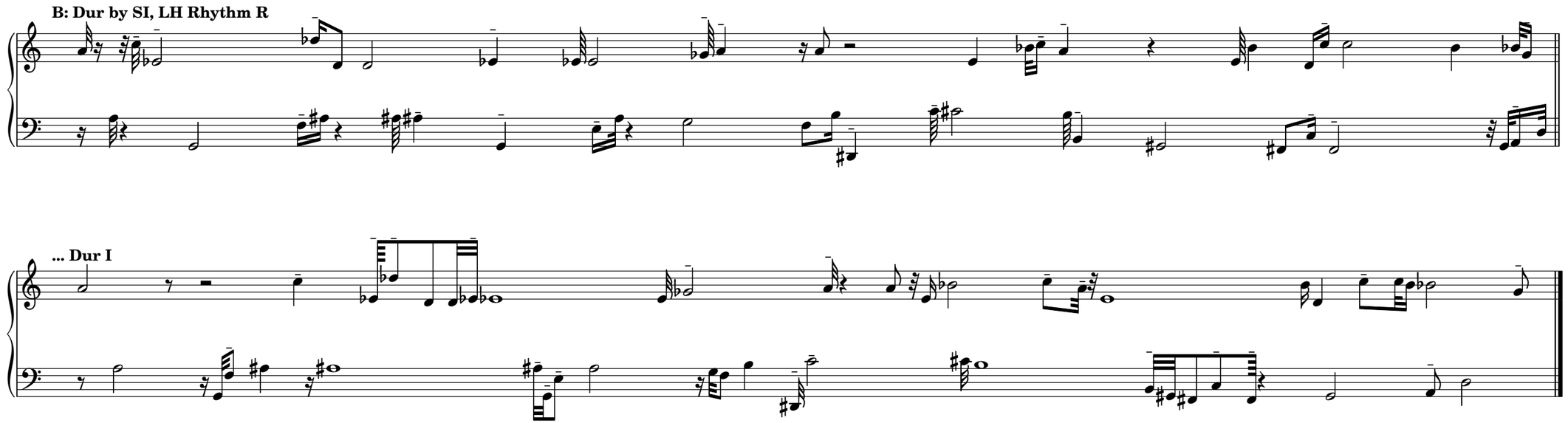
S7 **Octs [2 3-4]**
Seeds 6 11, Per 21 (F8)



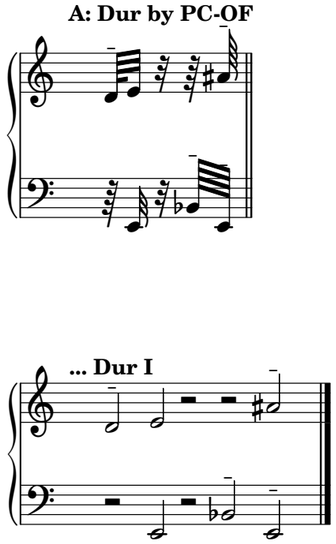


S8 **Octs [1-2 3]**
Seeds 7 12, Per 29 (L7)

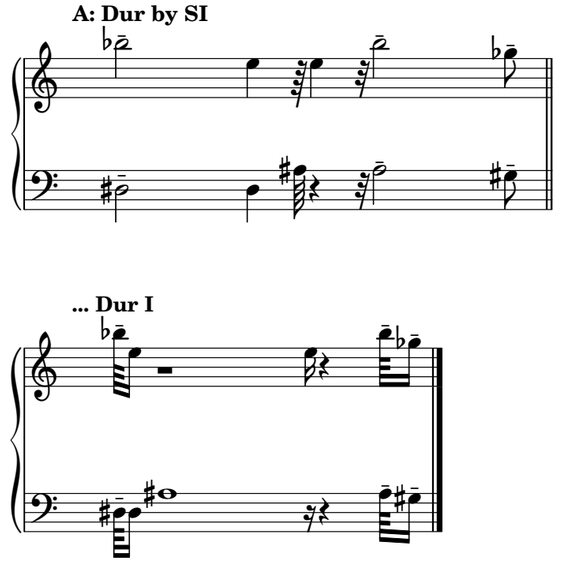


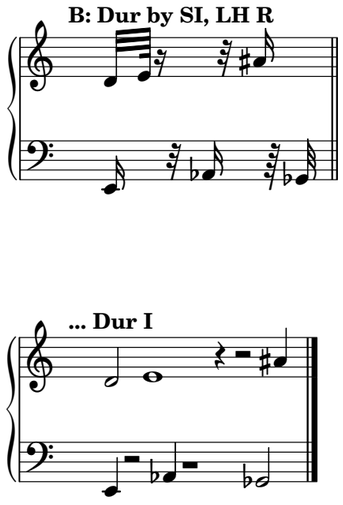


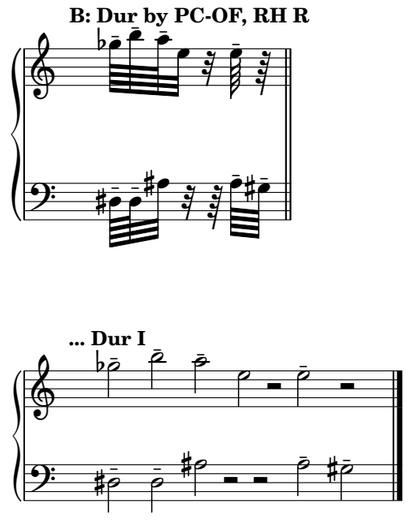
S9 Octs [1 3]
Seeds 8 14, Per 5 (F5)



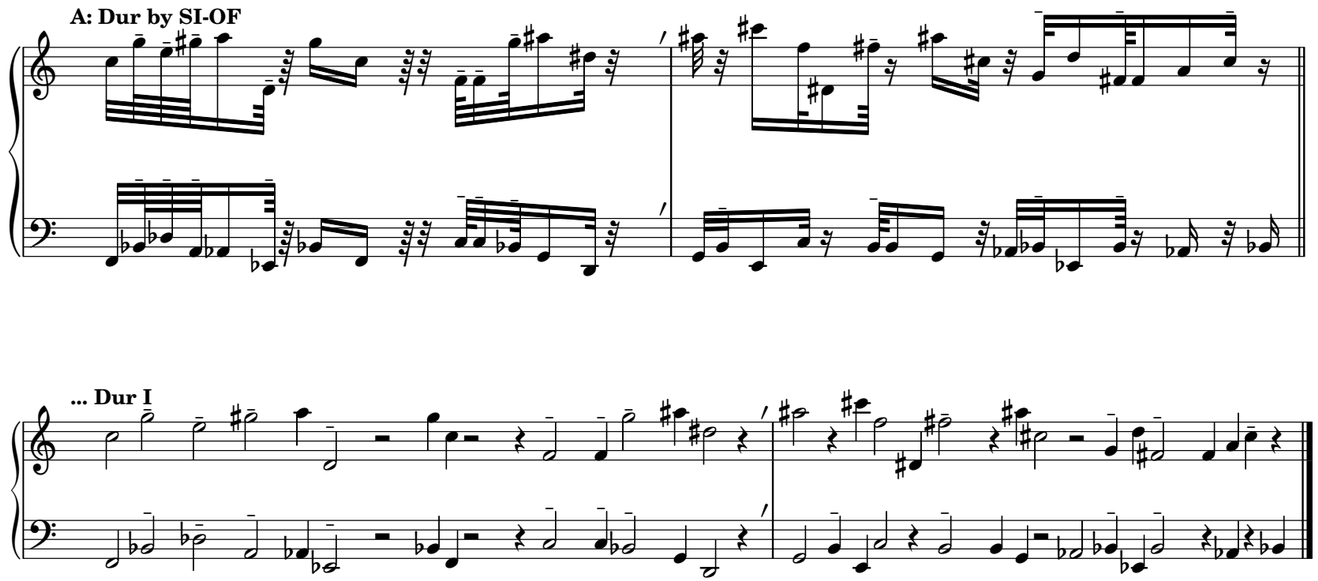
S10 Octs [2 4]
Seeds 9 16, Per 7 (L4)

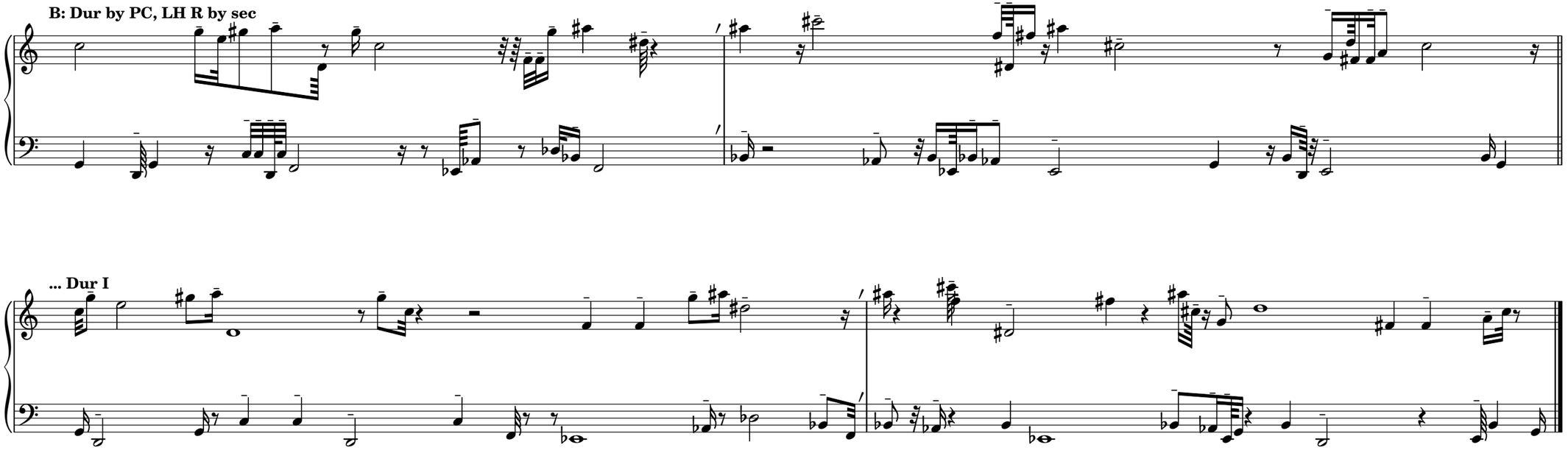






S11 **Octs [1 3-4]**
Seeds 10 17, Per 34 (F9)





S12 Octs [1-2 4]
Seeds 11 19, Per 46 (L8-1)

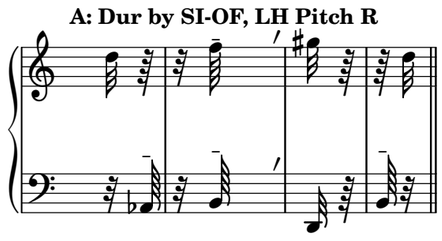
A: Dur by PC

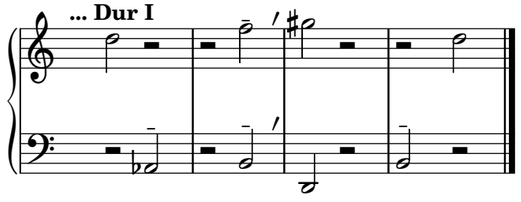
... Dur I

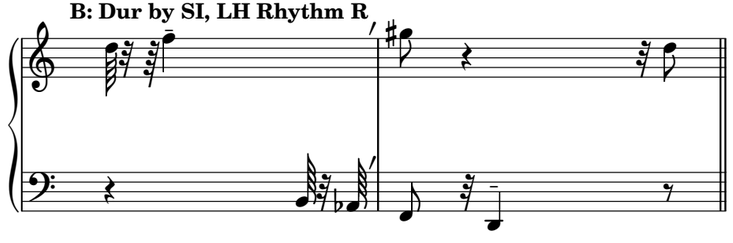
B: Dur by SI-OF, RH R by sec

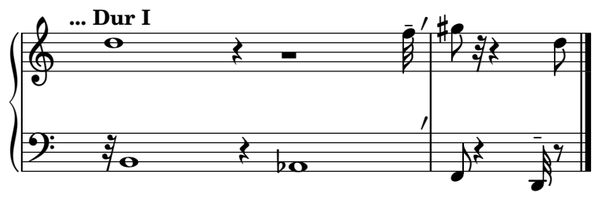
... Dur I

S13 Octs [1 4]
Seeds 12 21, Per 8 (F6)

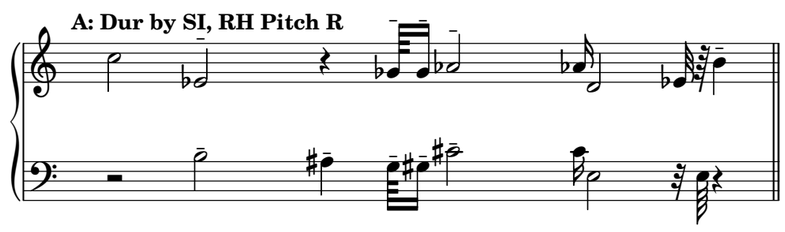


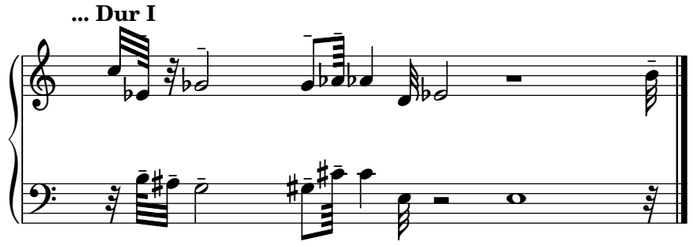


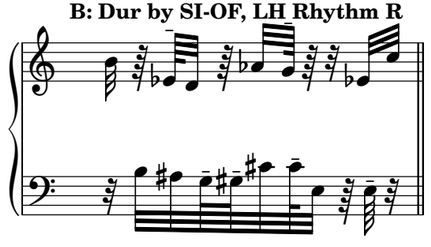


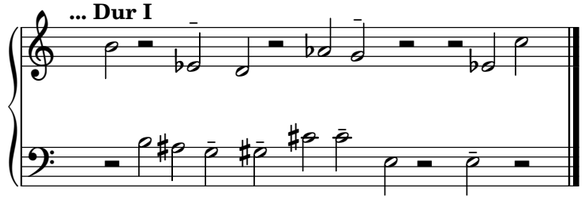


S14 Octs [2 3]
Seeds 13 22, Per 11 (L5)



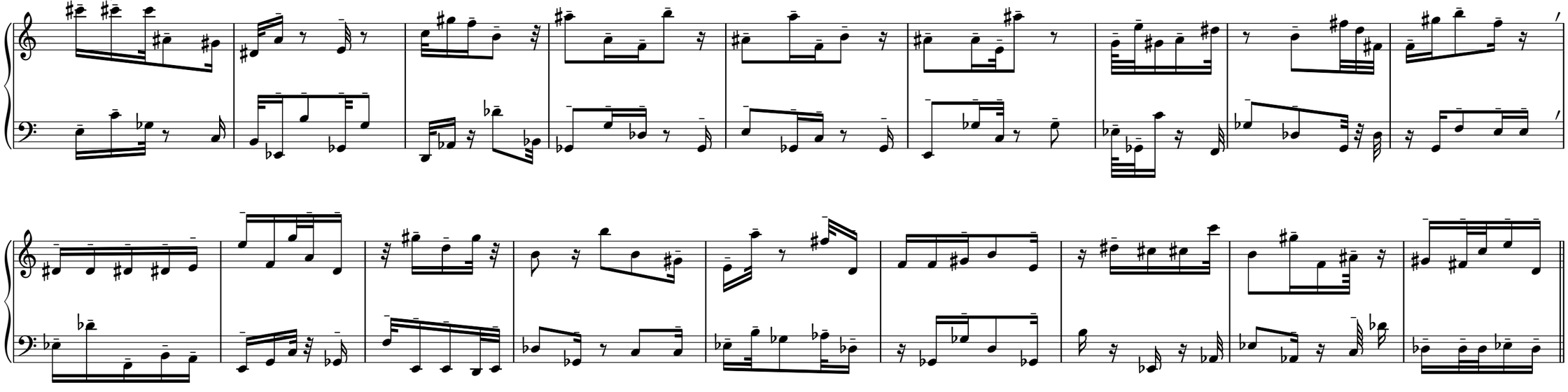




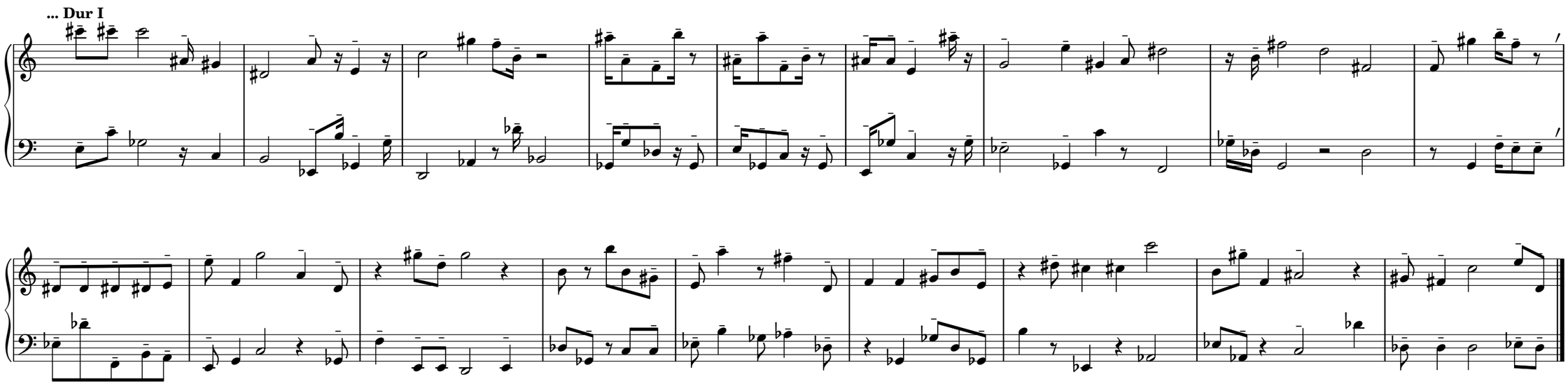


S15 Octs [1-2 3-4]
Seeds 14 24, Per 90 (F11+1)

A: Dur by PC-OF, LH Pitch R



... Dur I



B: Dur by PC, LH Rhythm R

... Dur I

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